

# Full counting statistics for $SU(N)$ impurity Anderson model

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We analyze the full counting statistics of a multiorbital Kondo effect in a quantum dot with the  $SU(N)$  symmetry in the framework of the renormalized perturbation theory. The current probability distribution function is calculated for an arbitrary dot-site Coulomb repulsion  $U$  in the particle-hole symmetric case. The resulting cumulant up to the leading nonlinear term of applied bias voltages indicates two types of electron transfer, respectively carrying charge  $e$  and  $2e$ , with different  $N$ -dependences. The cross correlation between different orbital currents shows exponential enhancement with respect to  $U$ , which directly addresses formation of the orbital-singlet state.

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Experimental realization of the Kondo state in mesoscopic devices has promoted further research of the Kondo effect since it enables one to access many-body effects in out-of-equilibrium under finite bias voltages.<sup>1,2</sup> Recent theoretical studies of nonequilibrium noise for Kondo dots<sup>3–9</sup> have predicted enhancement of shot noise with a fractional Fano factor, and have stimulated subsequent experimental studies.<sup>10–12</sup>

Deeper understanding of the nonequilibrium Kondo effect is brought in by the current distribution function, which includes higher-order cumulants beyond the first and second cumulant (average current and noise power). It, however, is still challenging to calculate the cumulant generating function (CGF) in the nonequilibrium Kondo states. Recently, Komnik, Gogolin and Schmidt have derived current probability distribution for the  $SU(2)$  Anderson impurity<sup>13–16</sup> from the general formulation of the full counting statistics (FCS).<sup>17,18</sup> They have clarified that the nonequilibrium backscattering current is composed of two types of electron transfer due to a single quasiparticle and a pair of quasiparticles, carrying charge  $e$  and  $2e$  respectively.

In the present work, we extend the FCS approach to multiorbital Kondo dots, which have been experimentally investigated in vertical dots,<sup>19</sup> carbon nanotubes,<sup>11</sup> and double dots.<sup>20</sup> We consider an  $SU(N)$  impurity Anderson model as a prototype model for examining multiorbital effects. By employing the renormalized perturbation theory (RPT)<sup>21,22</sup>, we calculate the zero-temperature CGF for the entire strength of the dot-site Coulomb repulsion. The RPT is based on a general idea of renormalization in quantum field theory, and is consistent with several known results, indicating its validity up to terms with third order of applied bias voltage.<sup>23,24</sup> Our calculation provides direct information on an *orbital singlet*, i.e., a correlated electronic state involving different orbitals.

*Model*— Let us consider a single quantum dot system described by the  $SU(N)$  impurity Anderson model  $\mathcal{H}_A = \mathcal{H}_0 + \mathcal{H}_T + \mathcal{H}_U$  with

$$\mathcal{H}_0 = \sum_{k\alpha m} \epsilon_{k\alpha m} c_{k\alpha m}^\dagger c_{k\alpha m} + \sum_m \epsilon_{dm} d_m^\dagger d_m, \quad (1)$$

$$\mathcal{H}_T = \sum_{k\alpha m} (v_\alpha d_m^\dagger c_{k\alpha m} + \text{H.c.}), \quad (2)$$

$$\mathcal{H}_U = \sum_{m < m'} U d_m^\dagger d_m d_{m'}^\dagger d_{m'}, \quad (3)$$

where  $d_m$  annihilates an electron in the dot level  $\epsilon_{dm}$  with orbital  $m = 1, 2, \dots, N$ ,  $c_{k\alpha m}$  annihilates a conduction electron with moment  $k$  and orbital  $m$  in lead  $\alpha = L, R$ , and  $U$  is the dot-site Coulomb repulsion. Here, orbital includes spin, and thus  $N$  is even. The intrinsic level width of the dot levels owing to tunnel coupling  $v_\alpha$ , is given by  $\Gamma = \sum_\alpha \pi \rho_c |v_\alpha|^2$  and the density of state of the conduction electrons  $\rho_c$ . For simplicity, the symmetric lead-dot coupling  $v_L = v_R$  and the particle-hole symmetry  $\epsilon_{dm} = -(N-1)U/2$  are assumed. The chemical potentials  $\mu_{L/R} = \pm V/2$ , satisfying  $\mu_L - \mu_R = V (\geq 0)$ , are measured relative to the Fermi level which is defined at zero voltage  $V = 0$ . We take a unit,  $\hbar = k_B = e = 1$ , throughout this paper.

*Full counting statistics*— The probability distribution  $P(\mathbf{q})$  of the transferred charge  $\mathbf{q} = (q_1, q_2, \dots, q_N)$  with orbital subscript across the dot during a time interval  $\mathcal{T}$  provides current correlation function of all orders. In this paper, we define the transferred charge operator as  $\hat{q}_m \equiv n_{Lm}(-\mathcal{T}/2) - n_{Lm}(\mathcal{T}/2)$ , where  $n_{Lm}(t)$  is the electron number in the left lead. In order to discuss the correlation functions systematically, we calculate the CGF  $\ln \chi(\boldsymbol{\lambda}) = \ln \sum_{\mathbf{q}} e^{i\boldsymbol{\lambda} \cdot \mathbf{q}} P(\mathbf{q})$  in the Keldysh formalism,<sup>25</sup>

$$\ln \chi(\boldsymbol{\lambda}) = \ln \langle T_C S_C^\lambda \rangle, \quad (4)$$

where  $S_C^\lambda = T_C \exp \{ -i \int_C dt [\mathcal{H}_T^\lambda(t) + \mathcal{H}_U(t)] \}$  is the time evolution operator for an extended Hamiltonian  $\mathcal{H}_A^\lambda = \mathcal{H}_0 + \mathcal{H}_T^\lambda + \mathcal{H}_U$ ,  $C$  is the Keldysh contour along  $[t : -\mathcal{T}/2 \rightarrow +\mathcal{T}/2 \rightarrow -\mathcal{T}/2]$ ,  $T_C$  is the contour ordering operator, and  $\boldsymbol{\lambda} = (\lambda_1, \lambda_2, \dots, \lambda_N)$  is the counting field. Here,  $\mathcal{H}_T^\lambda$  is given by

$$\mathcal{H}_T^\lambda = \sum_{km} \left[ v_L e^{i\lambda_m(t)/2} d_m^\dagger c_{kLm} + v_R d_m^\dagger c_{kRm} \right] + \text{H.c.} \quad (5)$$

with the contour dependent counting-field defined by  $\lambda_m(t) = \lambda_{m\mp} \equiv \pm \lambda_m$  for the forward and backward paths labeled, respectively, by “−” and “+”.

In order to calculate the CGF in Eq. (4), we make use of a procedure suggested by Komnik and Gogolin,<sup>26</sup> which is outlined below. First, a more general function  $\chi(\lambda_-, \lambda_+)$  with  $\lambda_{\mp} = (\lambda_{1\mp}, \lambda_{2\mp}, \dots, \lambda_{N\mp})$  is introduced. It is basically given by Eq. (4) but  $\lambda_{m\mp}$  is treated formally as an independent variable assigned for each contour. For the long time limit  $\mathcal{T} \rightarrow \infty$  where the switching effects are negligible, the general CGF is proportional to  $\mathcal{T}$ ,

$$\ln \chi(\lambda_-, \lambda_+) = -i\mathcal{T}\mathcal{U}(\lambda_-, \lambda_+), \quad (6)$$

with the adiabatic potential  $\mathcal{U}(\lambda_-, \lambda_+)$ . Once the adiabatic potential is computed, the statistics is recovered from  $\ln \chi(\lambda) = -i\mathcal{T}\mathcal{U}(\lambda, -\lambda)$ . Performing the derivative of Eqs. (4) and (6) with respect to  $\lambda_{m-}$ , we obtain

$$\frac{d}{d\lambda_{m-}} \mathcal{U}(\lambda_-, \lambda_+) = \lim_{\mathcal{T} \rightarrow \infty} \left\langle \frac{d}{d\lambda_{m-}} \mathcal{H}_T^\lambda(0_-) \right\rangle_{\lambda_-, \lambda_+}, \quad (7)$$

where we use a notation

$$\langle A(t) \rangle_{\lambda_-, \lambda_+} = \langle T_C S_C^\lambda A(t) \rangle / \chi(\lambda_-, \lambda_+). \quad (8)$$

Equation (8) presents an expectation for Hamiltonian  $\mathcal{H}_A^\lambda$  and the Wick's theorem is applicable. The right hand side of Eq. (7) can be expressed in terms of the Green's function,

$$\begin{aligned} & \frac{d}{d\lambda_{m-}} \mathcal{U}(\lambda_-, \lambda_+) \\ &= \frac{|v_L|^2}{2} \sum_k \int \frac{d\omega}{2\pi} \left[ e^{-i\bar{\lambda}_m/2} G_{dm}^{\lambda-+}(\omega) g_{Lkm}^{0+-}(\omega) \right. \\ & \quad \left. - e^{i\bar{\lambda}_m/2} g_{Lkm}^{0-+}(\omega) G_{dm}^{\lambda+-}(\omega) \right], \quad (9) \end{aligned}$$

with  $\bar{\lambda}_m = \lambda_{m-} - \lambda_{m+}$ .  $g_{k\alpha m}^{0-+}(\omega) = i2\pi\delta(\omega - \varepsilon_{k\alpha m})f_\alpha(\omega)$  and  $g_{k\alpha m}^{0+-}(\omega) = -i2\pi\delta(\omega - \varepsilon_{k\alpha m})[1 - f_\alpha(\omega)]$  are the lesser and greater parts of the Green's function for electrons in lead  $\alpha$  with the Fermi distribution function  $f_\alpha(\omega) = [e^{(\omega - \mu_\alpha)/T} + 1]^{-1}$ , respectively. For the long time limit  $\mathcal{T} \rightarrow \infty$ , the dot Green's function is defined as  $\{G_{dm}^\lambda(\omega)\}_{\nu\nu'} = -i \int d(t-t') e^{i\omega(t-t')} \langle T_C d_m(t_\nu) d_m^\dagger(t'_{\nu'}) \rangle_{\lambda_-, \lambda_+}$ . Here,  $\nu$  and  $\nu'$  are the labels for the two Keldysh contours.

*Renormalized perturbation theory*— The three basic parameters that specify the impurity Anderson model  $\mathcal{H}_A$  are  $\epsilon_{dm}$ ,  $\Gamma$  and  $U$ . The low-energy properties can be characterized by the quasiparticles with the renormalized dot-level  $\tilde{\epsilon}_{dm} = z[\epsilon_{dm} + \Sigma_{dm}^r(0)]$ , renormalized level width  $\tilde{\Gamma} = z\Gamma$ , and the renormalized interaction  $\tilde{U} = z^2\Gamma_{mm'}^{(4)}(0, 0; 0, 0)$  ( $m \neq m'$ ), where  $\Sigma_{dm}^r(\omega)$  is the self-energy of the retarded Green's function for the dot state:  $G_{dm}^r(\omega) = [\omega - \epsilon_{dm} + i\Gamma - \Sigma_{dm}^r(\omega)]^{-1}$ ,  $z = [1 - \partial\Sigma_{dm}^r(\omega)/\partial\omega|_{\omega=0}]^{-1}$  is the wave function renormalization factor, and  $\Gamma_{mm'}^{(4)}(\omega_1, \omega_2; \omega_3, \omega_4)$  is the local full four-point vertex function for the scattering of the electrons with the orbital  $m$  and  $m'$ .<sup>21,22</sup> Note that these parameters are defined at equilibrium  $V = 0$  and  $\lambda = 0$ .

The replacement of the bare parameters with the renormalized ones gives leading terms of the Hamiltonian corresponding to the low-energy fixed point of the Anderson model in Wilson's theory.<sup>27</sup> The perturbation theory in powers of  $U$  can be reorganized as an expansion with respect to the renormalized interaction  $\tilde{U}$ , taking the free quasiparticle Green's function  $\tilde{g}_{dm}^r(\omega) = (\omega - \tilde{\epsilon}_{dm} + i\tilde{\Gamma})^{-1}$  as the zero-order propagator. In addition, three counter terms are introduced in order to prevent overcounting. This procedure has enabled one to calculate the exact form of the Green's function in the absence of the counting fields  $\mathbf{G}_{dm}^\lambda(\omega)|_{\lambda=0}$  at low energies up to terms of order  $\omega^2$ ,  $V^2$ , and  $T^2$ .<sup>23,24</sup> There are explicit relations between the renormalized parameters and the enhancement factor of susceptibilities  $z\tilde{\chi}_d = 1 + \tilde{U}\tilde{\rho}_{dm}(0)$ ,  $z\tilde{\chi}_{c,d} = 1 - (N-1)\tilde{U}\tilde{\rho}_{dm}(0)$ , and the Friedel's sum rule  $\pi n_{dm} = \cot^{-1}(\tilde{\epsilon}_{dm}/\tilde{\Gamma})$  with the electron occupation in orbital  $m$  of the dot  $n_{dm} = \langle d_m^\dagger d_m \rangle$ , and the renormalized density of state  $\tilde{\rho}_{dm}(\omega) = (\tilde{\Gamma}/\pi)/[(\omega - \tilde{\epsilon}_{dm})^2 + \tilde{\Gamma}^2]$ . In particular, for the particle-hole symmetric case ( $n_{dm} = 1/2$ ), the renormalized parameters can be expressed in a simple form as,  $\tilde{U}/(\pi\tilde{\Gamma}) = R_N - 1$  and  $\tilde{\epsilon}_{dm} = 0$  with the Wilson ratio  $R_N = N/[(N-1) + \tilde{\chi}_{c,d}/\tilde{\chi}_d]$ .  $\tilde{\Gamma}$  can be considered as the Kondo temperature as,  $T_K = \pi\tilde{\Gamma}/4$ . In this paper, we evaluate the renormalized parameters by the Bethe ansatz exact solution (BAE)<sup>28,29</sup> and the numerical renormalization group (NRG) calculation<sup>27,30</sup>.

Let us now apply the RPT to calculation of the dot Green's function  $\mathbf{G}_{dm}^\lambda(\omega)$  for the extended Hamiltonian  $\mathcal{H}_A^\lambda$ . The dot Green's function is given by

$$\mathbf{G}_{dm}^\lambda(\omega) = z\tilde{\mathbf{G}}_{dm}^\lambda(\omega) = z \left[ \tilde{g}_{dm}^\lambda(\omega)^{-1} - \tilde{\Sigma}_{dm}^\lambda(\omega) \right]^{-1}, \quad (10)$$

where the zero-order part is given by<sup>15</sup>

$$\begin{aligned} \tilde{g}_{dm}^{\lambda--}(\omega) &= \left[ \omega + i\tilde{\Gamma}(f_L - 1/2) + i\tilde{\Gamma}(f_R - 1/2) \right] / \mathcal{D}_m, \\ \tilde{g}_{dm}^{\lambda-+}(\omega) &= \left[ ie^{i\bar{\lambda}_m/2}\tilde{\Gamma}f_L + i\tilde{\Gamma}f_R \right] / \mathcal{D}_m, \\ \tilde{g}_{dm}^{\lambda+-}(\omega) &= - \left[ ie^{i\bar{\lambda}_m/2}\tilde{\Gamma}(1 - f_L) + i\tilde{\Gamma}(1 - f_R) \right] / \mathcal{D}_m, \\ \tilde{g}_{dm}^{\lambda++}(\omega) &= \left[ -\omega + i\tilde{\Gamma}(f_L - 1/2) + i\tilde{\Gamma}(f_R - 1/2) \right] / \mathcal{D}_m, \end{aligned} \quad (11)$$

with

$$\begin{aligned} \mathcal{D}_m(\omega) &= \omega^2 + \tilde{\Gamma}^2 + \tilde{\Gamma}^2 \left[ \left( e^{-i\bar{\lambda}_m/2} - 1 \right) (1 - f_L) f_R \right. \\ & \quad \left. + \left( e^{i\bar{\lambda}_m/2} - 1 \right) (1 - f_R) f_L \right] \end{aligned} \quad (12)$$

The remainder part of the *renormalized* self-energy can be readily calculated in the second order perturbation in  $\tilde{U}$ , at  $T = 0$  up to  $\omega^2, \omega V$  and  $V^2$ , as

$$\Sigma_{dm}^\lambda(\omega) = \frac{-i}{8\tilde{\Gamma}} \left( \frac{\tilde{U}}{\pi\tilde{\Gamma}} \right)^2 \begin{bmatrix} A_m(\omega, V) & B_m(\omega, V) \\ -B_m^*(-\omega, -V) & A_m(\omega, V) \end{bmatrix} \quad (13)$$

with

$$A_m(\omega, V) = (N-1)[a(\omega, 3V/2) + 3a(\omega, V/2) + 3a(\omega, -V/2) + a(\omega, -3V/2)], \quad (14)$$

$$B_m(\omega, V) = \sum_{m'(\neq m)} \left\{ e^{-i\bar{\lambda}_{m'}/2} b(\omega, 3V/2) + \left[ 2 + e^{i(\bar{\lambda}_m - \bar{\lambda}_{m'})/2} \right] b(\omega, V/2) + \left[ 2e^{i\bar{\lambda}_m/2} + e^{i\bar{\lambda}_{m'}/2} \right] b(\omega, -V/2) + e^{i(\bar{\lambda}_m + \bar{\lambda}_{m'})/2} b(\omega, -3V/2) \right\} \quad (15)$$

Here,  $a(\omega, x) = -\frac{1}{2}(\omega - x)^2 \text{sgn}(-\omega + x)$ , and,  $b(\omega, x) = (\omega - x)^2 \theta(-\omega + x)$ . For  $\lambda = 0$ , the exact expression of usual self-energy up to  $\omega^2, \omega V$  and  $V^2$  is reproduced.<sup>9,23,24</sup> Substituting Eqs. (11) and (13) into Eq. (10), we readily obtain an expression for the Green's function up to  $\omega^2, \omega V$  and  $V^2$ .

*Results and discussion*— Substituting the obtained Green's function into Eq. (9) and integrating them respect with  $\lambda_{m-}$ , the CGF is derived at  $T = 0$  up to  $V^3$ , as  $\ln \chi(\lambda) = \mathcal{F}_0 + \mathcal{F}_1 + \mathcal{F}_2$  with

$$\mathcal{F}_0 = \frac{\mathcal{T}}{2\pi} \sum_m \int_{-V/2}^{V/2} d\omega \ln \left[ 1 + \frac{\tilde{\Gamma}^2}{\omega^2 + \tilde{\Gamma}^2} (e^{i\lambda_m} - 1) \right] \quad (16)$$

$$\mathcal{F}_1 = \frac{(N-1)\mathcal{T}}{24\pi} \frac{V^3}{\tilde{\Gamma}^2} (R_N - 1)^2 \sum_m (e^{-i\lambda_m} - 1), \quad (17)$$

$$\mathcal{F}_2 = \frac{\mathcal{T}}{6\pi} \frac{V^3}{\tilde{\Gamma}^2} (R_N - 1)^2 \sum_{(m \neq m')} \left[ e^{-i(\lambda_m + \lambda_{m'})} - 1 \right]. \quad (18)$$

Here,  $\sum_{(m \neq m')}$  takes sum of all combination of  $m$  and  $m'$  without  $m = m'$ .  $\mathcal{F}_0$  is the CGF of the zero-order part of the RPT,  $\mathcal{F}_1$  is the *single*-quasiparticle backscattering process carrying charge  $e$ , and  $\mathcal{F}_2$  is the *two*-quasiparticle backscattering process where the two quasiparticles with different orbitals make a singlet state, carrying charge  $2e$ . In particular,  $\mathcal{F}_1$  and  $\mathcal{F}_2$  represent the reflection of quasiparticles by the residual interaction  $\tilde{U}$ . This CGF for arbitrary  $U$  corresponds to the  $SU(N)$  extension of the hypothesis presented by Gogolin, Komnik and Schmidt.<sup>13,15,16</sup>

We now consider the cumulant for full current  $\mathcal{C}_n = (-i)^n \frac{d^n}{d\lambda^n} \ln \chi(\lambda)$ , which is derived from the CGF with  $\lambda_m = \lambda$  for all  $m$ , as

$$\mathcal{C}_n = \mathcal{T} [I_u \delta_{1n} + (-1)^n (P_{b0} + P_{b1} + 2^n P_{b2})], \quad (19)$$

where  $I_u = NV/(2\pi)$  is the linear-response current and  $\delta_{nn'}$  is the Kronecker's delta. We can obtain  $P_{b0} = \frac{N}{24\pi} \frac{V^3}{\tilde{\Gamma}^2}$  and  $P_{b1} = \frac{N(N-1)(R_N-1)^2}{24\pi} \frac{V^3}{\tilde{\Gamma}^2}$  from  $\mathcal{F}_0$  and  $\mathcal{F}_1$ , respectively. These express the probability of the single-quasiparticle backscattering processes carrying charge  $e$ , per time. Similarly, that of the two-quasiparticle backscattering process  $P_{b2} = \frac{N(N-1)(R_N-1)^2}{12\pi} \frac{V^3}{\tilde{\Gamma}^2}$  is obtained from  $\mathcal{F}_2$ . In this process, the two quasiparticles

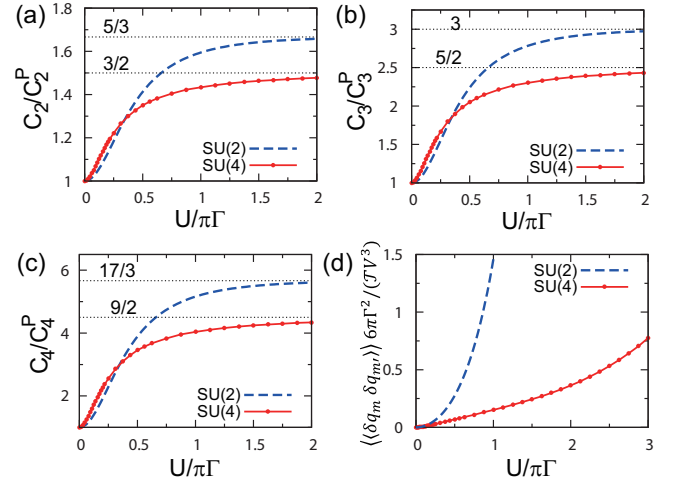


FIG. 1. (Color online) (a) The FFIR  $\mathcal{C}_n/\mathcal{C}_n^P$  for  $n = 2$ , (b)  $n = 3$ , and (c)  $n = 4$ , as a function of the Coulomb repulsion  $U$ . (d) The cross cumulant between fluctuation of transmitted charge with orbital  $m$  and  $m' (\neq m)$  as a function of the Coulomb repulsion  $U$ . The blue solid line and the red broken line present the  $SU(2)$  ( $N = 2$ ) case and the  $SU(4)$  ( $N = 4$ ) case, respectively.

with the different orbitals form a singlet pair and carry charge  $2e$ , which causes the factor  $2^n$  in Eq. (19).

In order to extract universal properties from the cumulant (19), we consider the Fano-factor *inspired* ratio (FFIR)<sup>13</sup> for  $n \geq 2$  and an arbitrary  $U$ , normalized in the form

$$\frac{\mathcal{C}_n}{\mathcal{C}_n^P} = \frac{1 + (1 + 2^{n+1})(N-1)(R_N-1)^2}{1 + 5(N-1)(R_N-1)^2}. \quad (20)$$

Here,  $\mathcal{C}_n^P = (-1)^n I_b \mathcal{T}$  ( $n \geq 2$ ) is the Poisson value of the CGF and  $I_b \equiv I_u - \mathcal{C}_1/\mathcal{T} = P_{b0} + P_{b1} + 2P_{b2}$  is the backscattering current. Thus, the FFIR can be interpreted as an average of the cumulants for the three backscattering current in this case. Remarkably, the FFIR is determined only by the two parameters, the Wilson ratio  $R_N$  and degeneracy  $N$ . The Fano factor which corresponds to the FFIR for  $n = 2$ , agrees with the previous works.<sup>7-9</sup> Evaluating  $R_N$  with the BAE for  $SU(2)$  ( $N = 2$ ),<sup>28,29</sup> and with the NRG for  $SU(4)$  ( $N = 4$ ),<sup>27,30</sup> we plot the FFIR for noise ( $n = 2$ ), skewness ( $n = 3$ ) and sharpness ( $n = 4$ ) as a function of the Coulomb repulsion  $U$  in Figs. 1(a)-1(c), respectively. In these figures, with increase of  $U$ , the FFIR crossovers from the Poisson value to the universal value of the  $SU(2)$  and  $SU(4)$  strong-coupling limit. Note that the  $n$  dependence, which enters through the factor  $2^n$  in Eq. (19), is caused by the two-quasiparticle process. It makes the FCS of the Kondo systems quite different from that of the noninteracting system  $U = 0$ . In the weak coupling limit  $U \rightarrow 0$ , the Wilson ratio takes the value of  $R_N = 1$ , and thus the FFIR goes to  $\mathcal{C}_n/\mathcal{C}_n^P \rightarrow 1$ . In the strong-coupling limit  $U \rightarrow \infty$ , the Wilson ratio approaches to the value  $R_N \rightarrow N/(N-1)$ , and the FFIR

TABLE I. The FFIR for  $n = 2, 3$  and 4 in the strong-coupling limit  $U \rightarrow \infty$ , given in Eq. (21), for several choices of degeneracy  $N$ .

$N$	2	4	6	8	$\rightarrow \infty$
$C_2/C_2^P$	5/3	3/2	7/5	4/3	$\rightarrow 1$
$C_3/C_3^P$	3	5/2	11/5	2	$\rightarrow 1$
$C_4/C_4^P$	17/3	9/2	19/5	10/3	$\rightarrow 1$

takes a universal form

$$C_n/C_n^P \rightarrow (N + 2^{n+1})/(N + 4). \quad (21)$$

The explicit values of the FFIR for several  $N$  are given in the TABLE I. For  $N = 2$ , Eq. (21) agrees with the result given by Gogolin and Komnik.<sup>13</sup> In the limit of large degeneracy  $N \rightarrow \infty$ , however, the FFIR approaches to the Poisson value  $C_n/C_n^P \rightarrow 1$ , even though the Coulomb repulsion has been taken first to be  $U \rightarrow \infty$ . This is because the renormalization becomes weaker for larger  $N$  and the two-quasiparticle process is suppressed  $P_{b2} \rightarrow 0$ .

We next consider cross cumulant which is observed as cross correlation between different orbital currents, and it may enable one to directly observe the two-quasiparticle scattering in experiments. The generic form of the cross cumulant between orbital currents is derived from the CGF up to terms of  $V^3$  as

$$\begin{aligned} \langle\langle \delta q_m^k \delta q_{m'}^l \rangle\rangle &= (-i)^{k+l} \frac{\partial^{k+l}}{\partial \lambda_m^k \partial \lambda_{m'}^l} \ln \chi(\boldsymbol{\lambda}) \\ &= (-1)^{k+l} \frac{\mathcal{T}}{6\pi} \frac{V^3}{\tilde{\Gamma}^2} (R_N - 1)^2, \end{aligned} \quad (22)$$

for  $k, l \geq 1$  and  $m \neq m'$ . The cross cumulant for  $(k, l) = (1, 1)$  is plotted as a function of the Coulomb repulsion  $U$  in Fig. 1 (d). In the strong-coupling region, the cross cumulant is inversely proportional to the square of the renormalized level width as  $\langle\langle \delta q_m^k \delta q_{m'}^l \rangle\rangle \propto 1/\tilde{\Gamma}^2$ , and, thus, increases exponentially with increase of the Coulomb repulsion as shown in Fig. 1 (d). The positive cross cumulant is a signature of orbital-singlet

states traveling through the dot. Larger orbital degeneracy makes renormalization weaker and suppresses the two-quasiparticle scattering. Therefore, the cross cumulant becomes the largest in the SU(2) case and the orbital degeneracy suppresses the correlation. We note that there is no higher order cross cumulant such as  $\langle\langle \delta q_m^j \delta q_{m'}^k \delta q_{m''}^l \rangle\rangle$  for  $m \neq m' \neq m''$  at low bias voltages determined by the terms up to order  $V^3$  in the particle-hole symmetric case even in the presence of the orbital degeneracy  $N > 2$ . Naively, it seems that the cross correlation can be observed in double dots with interdot Coulomb repulsion,<sup>20</sup> or dots connected to ferromagnetic leads with opposite polarizations.

Finally, we comment on cumulants of the current for orbital  $m$ ,  $C_n^m = (-i)^n \frac{d^n}{d\lambda_m^n} \ln \chi(\boldsymbol{\lambda}) = (-1)^n I_b^m$  with the backscattering current  $I_b^m = V/h - C_1^m$ . The cumulant  $C_n^m$  always takes the Poisson value  $C_n^m/C_n^{mP} = 1$  in the particle-hole symmetric case for the contribution up to  $V^3$  because there is no scattering of two quasiparticles with the same orbital in this case.

*Summary*— We have investigated the FCS of a multi-orbital Kondo dot described by the particle-hole symmetric SU( $N$ ) impurity Anderson model. Using the RPT, we derived the CGF for arbitrary Coulomb repulsion up to terms of order  $V^3$ . The dot-site Coulomb repulsion induces quasiparticle's orbital-singlet pairs carrying charge  $2e$  in the backscattering current. This process characterizes quantum fluctuations of the current in the correlated dot and is particularly manifest in the cross correlation between orbital current. It is also found that there is no electron entangled state carrying more than three quasiparticles in the particle-hole symmetric case in current up to order  $V^3$ , even in the presence of large orbital degeneracy  $N > 2$ .

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